Topics on Capital Misallocation

Lieyuan Yang

School of Economics, Renmin University of China

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Agenda for Today

- Accounting framework for capital misallocation (Hsieh and Klenow, 2009 QJE)
- Sources of misallocation

Basic Idea

- When resources are allocated efficiently

$$\text{marginal cost} = \text{marginal return}$$

- Consider a firm problem with Cobb-Douglas production function

$$\max_{k_i, n_i} z_i k_i^{\alpha} n_i^{1-\alpha} - Rk - wn$$

- First-order conditions

$$\alpha \frac{y_i}{k_i} = R, \qquad (1-\alpha)\frac{y_i}{n_i} = w$$

- Model implication
  - Firm's output-capital ratio should be proportional to interest rate R
  - Output-capital ratio should be equal across firms

However, in reality

- Firms differ in average product of capital
- Firm model with wedges

$$\max_{k_i,n_i}(1-\tau_{yi})z_i k_i^{\alpha} n_i^{1-\alpha} - (1+\tau_{ki})Rk - wn$$

- First-order condition

$$\frac{y_i}{k_i} = \frac{(1+\tau_{ki})R}{\alpha(1-\tau_{yi})}, \qquad \frac{y_i}{n_i} = \frac{w}{(1-\alpha)(1-\tau_{yi})}$$

- Model implication
  - Firms face distortions on capital rent and production. Higher wedges lead to higher marginal product.
  - If we can eliminate such distortions, we can promote aggregate productivity. How?

A Simple Model

- Consider a simple model of sectoral misallocation when the goods produced by two sectors are imperfect substitutes and there is a wedge.
- Suppose the final good comprises two groups (industries) of firms producing differentiated goods ("state and "private):

$$Y = Y_1^{1/2} Y_2^{1/2}$$

where

$$Y_1 = A_1 L_1 \quad \text{and} \quad Y_2 = A_2 L_2.$$

- Labor market clearing implies that $L_1 + L_2 = L$.
- Easy guess: Production efficiency requires that

$$L_1 = L_2 = L/2 \qquad \Rightarrow \qquad Y = A\frac{L}{2}$$

where $A \equiv A_1^{1/2} A_2^{1/2}$.

A Wedge

- Suppose production occurs in a market environment, and there are tax-like distortions.
- Firms' profits are given by

$$\Pi_1 = P_1 Y_1 - w(1+\tau_1)L_1$$
$$\Pi_2 = P_2 Y_2 - w(1+\tau_2)L_2$$

- Profit maximization yields

$$P_1 A_1 = w(1+\tau_1)$$
$$P_2 A_2 = w(1+\tau_2)$$

- We call $P_j A_j$ the Revenue TFP (RTFP).
- Note that, absent taxes, $\mathrm{RTFP}_1 = \mathrm{RTFP}_2$.
- With distortions, the more taxed sector has a higher RTFP than the less taxed sector.

Equilibrium

- The prices P_1 and P_2 are pinned down by their marginal product

$$P_1 = \frac{\partial Y}{\partial Y_1} = \frac{1}{2}\left(\frac{Y_2}{Y_1}\right)^{1/2} = \frac{1}{2}\left(\frac{A_2 L_2}{A_1 L_1}\right)^{1/2}, \quad P_2 = \frac{1}{2}\left(\frac{A_1 L_1}{A_2 L_2}\right)^{1/2}.$$

- Thus,

$$P_1 A_1 = \frac{A_1}{2}\left(\frac{A_2 L_2}{A_1 L_1}\right)^{1/2} = w(1+\tau_1)$$

$$P_2 A_2 = \frac{A_2}{2}\left(\frac{A_1 L_1}{A_2 L_2}\right)^{1/2} = w(1+\tau_2)$$

- Imposing the market clear condition $L_2 = L - L_1$ yields:

$$\frac{L - L_1}{L_1} = \frac{1+\tau_1}{1+\tau_2} \quad \Rightarrow \quad L_1 = \frac{1+\tau_2}{2+\tau_1+\tau_2} L, \quad L_2 = \frac{1+\tau_1}{2+\tau_1+\tau_2} L.$$

Government Budget Constraint

- Suppose the government subsidizes employment in one sector and taxes it in the other under a balanced budget constraint

$$w\tau_1 L_1 = -w\tau_2 L_2 \Rightarrow \tau_2 = -\tau_1 \frac{L_1}{L - L_1}$$

- Then:

$$L_1 = \frac{1+\tau_2}{2+\tau_1+\tau_2} L = \frac{1}{2(1+\tau_1)} L,$$
$$L_2 = \frac{1+\tau_1}{2+\tau_1+\tau_2} L = \frac{1+2\tau_1}{2(1+\tau_1)} L.$$

- If $\tau_1 > 0$, then $L_1 < L/2$ and $L_2 > L/2$.

Distorted Equilibrium

- If $\tau_1 = \tau_2 = 0$ (no distortions), employment is equally split among the two sectors:
$$L_1 = L_2 = \frac{1}{2}L$$
- When Sector 1 is taxed (subsidized), L_1 becomes inefficiently too small (too large) relative to L_2.
- Empirically, $\tau_1 > \tau_2$ shows up as sector 2 having a relatively large labor-income share:
$$1 + \tau_1 = \frac{P_1 Y_1}{wL_1} > 1 > \frac{P_2 Y_2}{wL_2} = 1 + \tau_2$$

Distorted Equilibrium

- Consider GDP:

$$\begin{aligned} Y &= (A_1 L_1)^{1/2} (A_2 L_2)^{1/2} \\ &= A \left[\frac{1}{2(1+\tau_1)} \right]^{1/2} \left[\frac{1+2\tau_1}{2(1+\tau_1)} \right]^{1/2} L \\ &= A \frac{(1+2\tau_1)^{1/2}}{2(1+\tau_1)} L, \end{aligned}$$

  where $A \equiv A_1^{1/2} A_2^{1/2}$.

- Wages are equal to

$$w = A \frac{(1+2\tau_1)^{1/2}}{2(1+\tau_1)}.$$

Model of Monopolistic Competition: Final Goods

- A model of monopolistic competition.
- Firms are assumed to differ in their efficiency levels and potentially face different output and capital distortions (wedges).
- A representative firm in a competitive final output market produces the single final good Y.
- Y is produced by combining the output Y_s of S manufacturing industries using a Cobb-Douglas production technology:

$$Y = \prod_{s=1}^{S} Y_s^{\theta_s}, \qquad \sum_{s=1}^{S} \theta_s = 1.$$

- Notation: P_s is the price of industry output Y_s and $P \equiv \prod_{s=1}^{S} (P_s/\theta_s)^{\theta_s}$ is the price of the final good.
- The final good is chosen to be the numeraire, $P = 1$.
- Cost minimization implies $P_s Y_s = \theta_s PY$

- Industry output Y_s is a CES aggregate of M_s differentiated products:

$$Y_s = \left(\sum_{i=1}^{M_s} Y_{si}^{(\sigma-1)/\sigma} \right)^{\sigma/(\sigma-1)}, \quad \sigma > 1$$

- The production function for each differentiated product is given by a Cobb-Douglas function of firm TFP, capital, and labor:

$$Y_{si} = A_{si} K_{si}^{\alpha_s} L_{si}^{1-\alpha_s},$$

where capital's share of output α_s is the same for all firms within an industry s, but may differ across industries.

Firm Demand under Monopolistic Competition

Solving the final-good producer's problem

Recall that $Y = \prod_s Y_s^{\theta_s}$ and $Y_s = (\sum_i Y_{si}^{(\sigma-1)/\sigma})^{\sigma/(\sigma-1)}$. Given a budget constraint

$$B = \sum_s \sum_i P_{si} Y_{si},$$

the final good producer's problem can be expressed as

$$\Lambda = \prod_{s=1}^{S} \left(\sum_{i=1}^{M_s} Y_{si}^{(\sigma-1)/\sigma} \right)^{\sigma\theta_s/(\sigma-1)} + \lambda \left(B - \sum_s \sum_i P_{si} Y_{si} \right).$$

Taking the FOC for firm i in sector J yields the firm demand function,

$$\frac{\partial \Lambda}{\partial Y_{Ji}} = \theta_J Y \left(\sum_{j=1}^{M_J} Y_{Jj}^{(\sigma-1)/\sigma} \right)^{-1} Y_{Ji}^{-1/\sigma} - \lambda P_{Ji} = 0$$

$$\Rightarrow P_{Ji} = Y_{Ji}^{-1/\sigma} \frac{\theta_J Y}{\lambda Y_J^{(\sigma-1)/\sigma}}.$$

Model of Monopolistic Competition

Distortions and Profits

- In principle, there might be distortions to labor, capital, and output. But three distortions cannot be separately identified. Because this framework includes two factors of production, two separate distortions can be identified:
  1. Normalize the labor distortion to zero: $\tau_L = 0$.
  2. Output distortions τ_Y that increase the marginal products of capital and labor by the same proportion.
  3. Capital distortions τ_K that increase the marginal product of capital relative to labor.
- Profits are given by:

$$\pi_{si} = (1 - \tau_{Ysi}) P_{si} Y_{si} - wL_{si} - (1 + \tau_{Ksi}) RK_{si}$$

- All firms are assumed to face the same wage.

- Recall that demand function for monopolist is

$$P_{si} = \chi_s Y_{si}^{-1/\sigma}$$

where χ_s is a constant which depends on P_s.

- Rewrite the profit maximization as

$$\begin{aligned}\max_{K_{si},L_{si}} \pi_{si} &= (1-\tau_{Ysi})P_{si}Y_{si} - wL_{si} - (1+\tau_{Ksi})RK_{si} \\ &= (1-\tau_{Ysi})\chi_s \left(A_{si}K_{si}^{\alpha_s}L_{si}^{1-\alpha_s}\right)^{1-1/\sigma} \\ &\quad - wL_{si} - (1+\tau_{Ksi})RK_{si}.\end{aligned}$$

Model of Monopolistic Competition

Profit Maximization 2

- Profit maximization: take FOCs w.r.t. L_{si} and K_{si}

$$(1-\alpha_s)\frac{\sigma-1}{\sigma}(1-\tau_{Ysi})\frac{P_{si}Y_{si}}{L_{si}} = w$$
$$\alpha_s\frac{\sigma-1}{\sigma}(1-\tau_{Ysi})\frac{P_{si}Y_{si}}{K_{si}} = (1+\tau_{Ksi})R$$

- ... yields the standard condition that the firm's output price is a fixed markup over its marginal cost:

$$P_{si} = \underbrace{\frac{\sigma}{\sigma-1}}_{\text{markup}}\underbrace{\left(\frac{R}{\alpha_s}\right)^{\alpha_s}\left(\frac{w}{1-\alpha_s}\right)^{1-\alpha_s}\frac{(1+\tau_{Ksi})^{\alpha_s}}{A_{si}(1-\tau_{Ysi})}}_{\text{marginal cost of producing 1 unit } Y_{si}}$$

- The capital-labor ratio, labor allocation, and output of each firm are given by:

$$\frac{K_{si}}{L_{si}} = \frac{\alpha_s}{1-\alpha_s}\frac{w}{R}\frac{1}{1+\tau_{Ksi}}$$

$$L_{si} \propto A_{si}^{\sigma-1}\frac{(1-\tau_{Ysi})^{\sigma}}{(1+\tau_{Ksi})^{\alpha_s(\sigma-1)}}$$

$$Y_{si} \propto A_{si}^{\sigma}\left[\frac{1-\tau_{Ysi}}{(1+\tau_{Ksi})^{\alpha_s}}\right]^{\sigma}$$

- The allocation of resources across firms depends not only on firm TFP levels but also on firm output and capital distortions, τ_Y and τ_K.

- The marginal revenue product of labor is proportional to revenue per worker and is given by:

$$\mathrm{MRPL}_{si} \equiv (1-\alpha_s)\frac{\sigma-1}{\sigma}\frac{P_{si}Y_{si}}{L_{si}} = \frac{w}{1-\tau_{Ysi}}$$

- The marginal revenue product of capital is proportional to the revenue-capital ratio and is given by:

$$\mathrm{MRPK}_{si} \equiv \alpha_s\frac{\sigma-1}{\sigma}\frac{P_{si}Y_{si}}{K_{si}} = R\frac{1+\tau_{Ksi}}{1-\tau_{Ysi}}.$$

- Solving for the equilibrium allocation of resources across sectors yields:

$$L_s \equiv \sum_i L_{si} = L \frac{(1-\alpha_s)\theta_s / \overline{\mathrm{MRPL}}_s}{\sum_{s'=1}^{S} (1-\alpha_{s'})\theta_{s'} / \overline{\mathrm{MRPL}}_{s'}},$$

$$K_s \equiv \sum_i K_{si} = K \frac{\alpha_s \theta_s / \overline{\mathrm{MRPK}}_s}{\sum_{s'=1}^{S} \alpha_{s'}\theta_{s'} / \overline{\mathrm{MRPK}}_{s'}}.$$

where $L \equiv \sum_s L_s$, $K \equiv \sum_s K_s$, and

$$\overline{\mathrm{MRPL}}_s = \frac{w}{\sum_i (1-\tau_{Ysi}) \frac{P_{si}Y_{si}}{P_s Y_s}}, \quad \overline{\mathrm{MRPK}}_s = \frac{R}{\sum_i \frac{1-\tau_{Ysi}}{1+\tau_{Ksi}} \frac{P_{si}Y_{si}}{P_s Y_s}}$$

are the weighted average MRPL and MRPK in a sector.

- Take a case in which there are no differences in α_s nor in θ's:

$$L_s \equiv \sum_i L_{si} = L \frac{1/\overline{\mathrm{MRPL}}_s}{\sum_{s'=1}^{S} 1/\overline{\mathrm{MRPL}}_{s'}},$$

$$K_s \equiv \sum_i K_{si} = K \frac{1/\overline{\mathrm{MRPK}}_s}{\sum_{s'=1}^{S} 1/\overline{\mathrm{MRPK}}_{s'}}.$$

- A sector with high MRPL employs fewer workers.
- A sector with high MRPK employs less capital.

- Aggregate Y can be expressed as a function of K_s, L_s, and industry TFP:

$$Y = \prod_{s=1}^{S} \left(\text{TFP}_s K_s^{\alpha_s} L_s^{1-\alpha_s}\right)^{\theta_s}$$

- Suppose again same α_s and θ_s:

$$\begin{aligned} Y &= \prod_{s=1}^{S} (\text{TFP}_s K_s^{\alpha} L_s^{1-\alpha})^{\theta} \\ &= K^{\alpha} L^{1-\alpha} \prod_{s=1}^{S} \left[\text{TFP}_s \left(\frac{1/\overline{\text{MRPK}}_s}{\sum_{s'} 1/\overline{\text{MRPK}}_{s'}} \right)^{\alpha} \left(\frac{1/\overline{\text{MRPL}}_s}{\sum_{s'} 1/\overline{\text{MRPL}}_{s'}} \right)^{1-\alpha} \right]^{\theta}. \end{aligned}$$

Model of Monopolistic Competition

Physical Productivity vs. Revenue Productivity

- TFPR (revenue productivity) is the product of physical productivity times output price:

$$\text{TFPR}_{si} \equiv P_{si} A_{si} = \frac{P_{si} Y_{si}}{K_{si}^{\alpha_s} L_{si}^{1-\alpha_s}}$$

- TFP (physical productivity) is defined as:

$$\text{TFP}_{si} \equiv A_{si} = \frac{Y_{si}}{K_{si}^{\alpha_s} L_{si}^{1-\alpha_s}}$$

- We often have data on $P_{si} Y_{si}$ but not separately on P_{si} and Y_{si}. Very useful to work with TFPR.

Model of Monopolistic Competition

Physical Productivity vs. Revenue Productivity

- In the absence of distortions, TFPR does not vary across plants within an industry. Therefore, more capital and labor should be allocated to plants with higher TFP until their higher output results in a lower price and the exact same TFPR as at smaller plants.
- Using the definitions of the marginal revenue products of capital and labor, plant TFPR is proportional to a geometric average of plant MRPK and MRPL:

$$\mathrm{TFPR}_{si} \propto \mathrm{MRPK}_{si}^{\alpha_s} \mathrm{MRPL}_{si}^{1-\alpha_s} \propto \frac{(1+\tau_{Ksi})^{\alpha_s}}{1-\tau_{Ysi}}$$

- High plant TFPR shows the presence of barriers that raise the plant's marginal products of capital and labor, rendering the plant smaller than optimal.

Model of Monopolistic Competition

Industry TFP

- If marginal products were equalized across plants, industry TFP would be

$$\overline{A}_s = \left(\sum_{i=1}^{M_s} A_{si}^{\sigma-1} \right)^{1/(\sigma-1)}.$$

- With distortions, it is a function of the misallocation of capital and labor:

$$\mathrm{TFP}_s = \left[\sum_{i=1}^{M_s} \left(A_{si} \frac{\overline{\mathrm{TFPR}_s}}{\mathrm{TFPR}_{si}} \right)^{\sigma-1} \right]^{1/(\sigma-1)},$$

where $\overline{\mathrm{TFPR}_s} \propto \overline{\mathrm{MRPK}_s}^{\alpha_s} \overline{\mathrm{MRPL}_s}^{1-\alpha_s}$ is a geometric average of the average marginal revenue product of capital and labor in the sector.

- This equation is used for the empirical estimates.

Model of Monopolistic Competition

Special Case: TFPQ and TFPR are Jointly Log-Normally Distributed

- When TFP and TFPR are jointly log-normally distributed, there exists a simple closed-form expression for aggregate TFP:

$$\log \mathrm{TFP}_s = \frac{1}{\sigma - 1} \log \left(\sum_{i=1}^{M_s} A_{si}^{\sigma - 1} \right) - \frac{\sigma}{2} \operatorname{var}(\log \mathrm{TFPR}_{si})$$

- Here, the negative effects of distortions on aggregate TFP can be summarized by the variance of log TFPR.
- Intuitively, the extent of misallocation is worse when there is greater dispersion of marginal products.
- Note that the correlation between TFP and TFPR (and, hence, the wedges) is irrelevant. Reason: log-normal distribution.

Quantitative Analysis

- Dataset
  - India's census of all registered manufacturing plants.
  - China's Annual Surveys of Industrial Production conducted by the National Bureau of Statistics.
  - U.S. Census of Manufacturers covering all manufacturing plants.
- Key Parameters for Estimation
  - The rental price of capital is set to $R = 0.10$ (5% real interest rate and 5% depreciation rate). The actual cost of capital faced by plant i in industry s is $(1 + \tau_{Ksi})R$.
  - The elasticity of substitution between plant value-added is set to $\sigma = 3$ (robustness check with $\sigma = 5$). Having a single σ across all goods is a strong assumption.
  - Because the U.S. is assumed to be comparatively undistorted, industry capital shares are set to those in the corresponding U.S. manufacturing industry.

Potential Gains from Reallocation

Inference of Plant Distortions and TFP

- Given all parameters and the plant data, distortions for each plant in each country-year are inferred as follows:
- A capital distortion is inferred when the ratio of labor compensation to the capital stock is "high:

$$1+\tau_{Ksi}=\frac{\alpha_s}{1-\alpha_s}\frac{wL_{si}}{RK_{si}}$$

- An output distortion is deduced when labor's share is "low":

$$1-\tau_{Ysi}=\frac{\sigma}{\sigma-1}\frac{wL_{si}}{(1-\alpha_s)P_{si}Y_{si}}$$

- Given all parameters and the plant data, plant productivity in each country-year can be inferred as follows:

$$A_{si}=\kappa_s\frac{(P_{si}Y_{si})^{\sigma/(\sigma-1)}}{K_{si}^{\alpha_s}L_{si}^{1-\alpha_s}}$$

Potential Gains from Reallocation

Distribution of TFPR

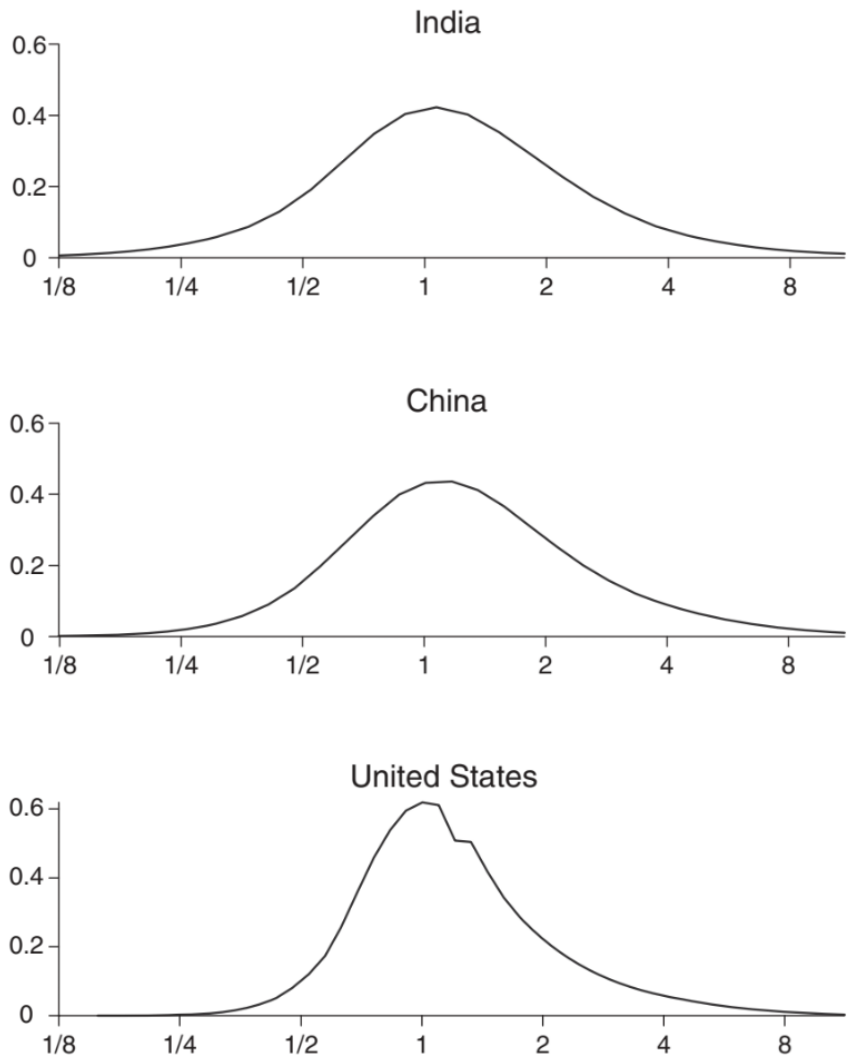


FIGURE II
Distribution of TFPR

Potential Gains from Reallocation

Distribution of TFPR

- The figure plots the distribution of TFPR, $\log(\mathrm{TFPR}_{si}/\overline{\mathrm{TFPR}_s})$, for the latest year in each country.
- TFPR dispersion is higher in India than in the U.S.
- Despite not fully sampling small private establishments, China also exhibits greater TFPR dispersion than the United States.
- The dispersion statistics are consistent with greater distortions in China and India than the United States.

Potential Gains from Reallocation

Efficient Output

- If marginal products were equalized across plants in a given industry, then industry TFP would be

$$\overline{A}_s = \left(\sum_{i=1}^{M_s} A_{si}^{\sigma-1} \right)^{1/(\sigma-1)}.$$

- For each industry, the ratio of actual TFP to this efficient level of TFP is calculated, and then aggregated across sectors using the Cobb-Douglas aggregator:

$$\frac{Y}{Y_{\text{efficient}}} = \prod_{s=1}^{S} \left[\sum_{i=1}^{M_s} \left(\frac{A_{si}}{\overline{A}_s} \frac{\overline{\text{TFPR}_s}}{\text{TFPR}_{si}} \right)^{\sigma-1} \right]^{\theta_s/(\sigma-1)}$$

Potential Gains from Reallocation

Gains from Equalizing TFPR

TABLE IV

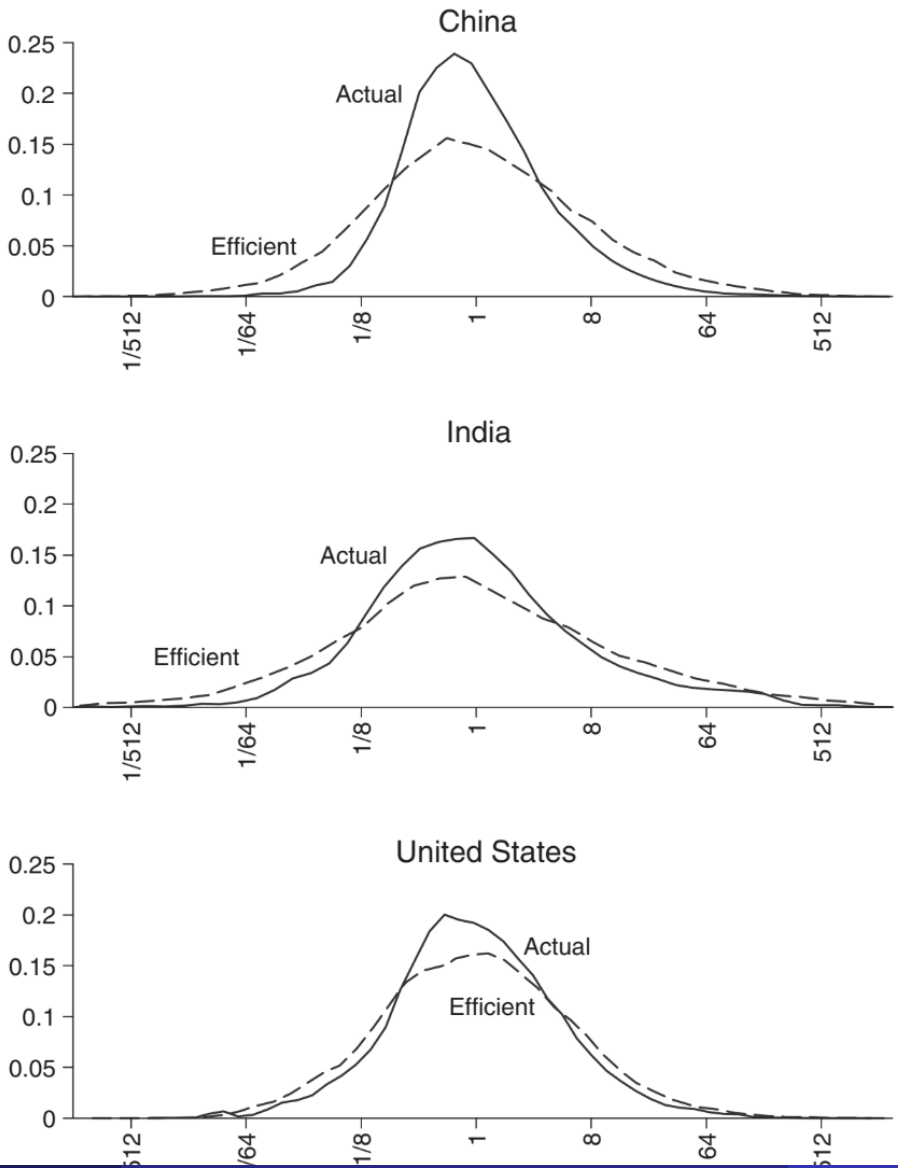
TFP GAINS FROM EQUALIZING TFPR WITHIN INDUSTRIES

China	1998	2001	2005
%	115.1	95.8	86.6
India	1987	1991	1994
%	100.4	102.1	127.5
United States	1977	1987	1997
%	36.1	30.7	42.9

Notes. Entries are $100(Y_{\text{efficient}}/Y - 1)$ where $Y/Y_{\text{efficient}} = \prod_{s=1}^{S}[\sum_{i=1}^{M_s}(\frac{A_{si}}{\bar{A}_s}\frac{\overline{\text{TFPR}}_s}{\text{TFPR}_{si}})^{\sigma-1}]^{\theta_s/(\sigma-1)}$ and $\text{TFPR}_{si} \equiv \frac{P_{si}Y_{si}}{K_{si}^{\alpha_s}(w_{si}L_{si})^{1-\alpha_s}}$.

Potential Gains from Reallocation

Efficient vs. Actual Size Distribution of Plants



Potential Gains from Reallocation

Efficient vs. Actual Size Distribution of Plants

- The figure plots the "efficient" vs. actual size distribution of plants in the latest year.
- Size is measured as plant value-added.
- In all three countries, the hypothetical efficient distribution is more dispersed than the actual one.
- There should be fewer mid-sized plants and more small and large plants.
- In China and India, initially large plants are less likely to shrink and more likely to expand than in the U.S when we move to efficiency.
- Thus, TFPR increases with size more strongly in China and India.

Potential Gains from Reallocation

Percent TFP Gains in China and India Relative to the U.S.

TABLE VI
TFP GAINS FROM EQUALIZING TFPR RELATIVE TO 1997 U.S. GAINS

China	1998	2001	2005
%	50.5	37.0	30.5
India	1987	1991	1994
%	40.2	41.4	59.2

Potential Gains from Reallocation

Percent TFP Gains in China and India Relative to the U.S.

- Hypothetically moving to U.S. efficiency could have boosted Chinese TFP by 50% in 1998, 37% in 2001, and 30% in 2005.
- Compared to the 1997 U.S. benchmark, Chinese allocative efficiency improved by 15% from 1998 to 2005 (2% per year).
- For India, hypothetically moving to U.S. efficiency could have raised TFP by 40% in 1987 or 1991, and 59% in 1994.
- There is no evidence of improving allocations in India from 1987 to 1994. The implied decline in allocative efficiency of 12% (1.8% per year) is surprising, given that many Indian reforms began in the late 1980s.

Measurement Error

- Estimated potential efficiency gains could result from greater measurement error in Chinese and Indian data than in U.S. data.
- While arbitrary measurement error cannot be ruled out, Hsieh and Klenow try to evaluate whether the results can be attributed to specific forms of measurement error (such as extreme outliers, correlation of TFPR and TFPQ with plant exit, measurement error in plant revenue and inputs, etc.)
- The results are inconclusive. While there is no clear evidence for higher measurement error in China and India than in the U.S., it cannot be ruled out as the source of the greater TFPR dispersion.

Policies and Misallocation

Explanations for TFPR Dispersion

- If TFPR dispersion is real rather than a by-product of measurement error, TFPR gaps can be related to explicit government policies.
- In China, TFPR dispersion can be related to state ownership of plants.
- In India, TFPR dispersion can be related to licensing and size restrictions.

Policies and Misallocation

State Ownership of Plants in China

- The percentage of state-owned plants in China has fallen dramatically over the past years (29% in 1998, 19% in 2001, 8% in 2005).
- TFPR is roughly 40% lower at state-owned plants than at private domestic plants.
- How much of China's TFPR dispersion can be accounted for by state ownership?

TABLE XII
REGRESSIONS OF SECTOR TFPR DISPERSION ON STATE OWNERSHIP IN CHINA

	1998	2001	2005	1998–2005
State ownership	0.766	0.659	0.025	0.300
share	(0.165)	(0.153)	(0.213)	(0.080)
Year F.E.	NO	NO	NO	YES
Sector F.E.	NO	NO	NO	YES
N	406	403	407	3,237

Note. Entries are coefficients from regressions of the variance of log TFPR in sector s on the variance in sector s of an indicator variable for a state-owned plant. All regressions are weighted by the value-added weights of the sector. Standard errors are clustered by sector in the last column.

- In India, distortions from licensing and size restrictions may prevent efficient plants from achieving optimal scale and keep inefficient plants from contracting or exiting.
- There were two waves of delicensing of industries by the Indian government: in 1985 and in 1991.
- Size restrictions were lifted much later (1997-2005) and are not in the sample.
- Relating the dispersion of industry TFPR to whether the industry was delicensed in 1991 and whether the industry faced size restrictions (and including a dummy for industries delicensed in 1985) yields the following results.

TABLE XIII
REGRESSION OF SECTOR TFPR DISPERSION ON DELICENSING AND SIZE RESTRICTIONS IN INDIA

	(1)	(2)	(3)
Delicensed 1991	−0.298		−0.298
	(0.117)		(0.117)
Delicensed 1991 × post-1991	0.032		−0.056
	(0.036)		(0.040)
Size restriction		0.368	
		(0.173)	
Delicensed 1991 × post 1991 × size restriction			0.415
			(0.120)

Notes. The dependent variable is the variance of log TFPR in sector s in year t. Entries are coefficients on the following independent variables: (1) delicensed 1991: indicator for whether industry was delicensed in 1991; (2) delicensed 1991 × post 1991: product of an indicator for an industry delicensed in 1991 and an indicator for observations after 1991; (3) size restriction: % of value-added of an industry subject to reservations for small firms and; (4) delicensed 1991 × post 1991 × size restriction: product of size restriction, indicator variable for observations after 1991, and a dummy variable for industries delicensed after 1991. All regressions include indicator variables for year (1987 through 1994) and are weighted by the value-added share of the sector. Regressions (1) and (3) also include a dummy for industries delicensed in 1985. The omitted group consists of industries not delicensed in either 1985 or 1991. Standard errors are clustered by sector. Number of observations = 2,644.

Extension

- What other factors can generate capital misallocation?
- Does measured capital misallocation necessarily reflect inefficient allocation?
- Beyond capital, what other resources can be misallocated?

CES Production Function

- Monopolistic competition

$$Y = \left(\int_0^1 y_i^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}}$$

- Production function

$$y_i = \left[\alpha (A_{k,i} k_i)^{\frac{\sigma-1}{\sigma}} + (1-\alpha)(A_{n,i} n_i)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}$$

- Firms' problem

$$\max_{p,k,n} p_i y_i - R k_i - w n_i$$

- Optimal allocation

$$\frac{p_i y_i}{k_i} = \frac{\varepsilon}{\varepsilon - 1} R \left[1 + \left(\frac{1-\alpha}{\alpha} \right)^{\sigma} \left(\frac{w}{R} \right)^{1-\sigma} \left(\frac{A_{k,i}}{A_{n,i}} \right)^{1-\sigma} \right]$$

- Dispersion in revenue per unit of capital need not imply dispersion in the marginal revenue product of capital.

Dynamic Inputs (ACD 2014 JPE; DV 2019 AER)

- Firm dynamic model

$$v(k,b,z) = \max_{p,n,k',c,b'} \left\{ u(c) + \beta \mathbb{E} v(k',b',z') \right\}$$

$$\text{s.t.} \quad c + k' + (1+r)b + \mathcal{C}(k,k') = py - wn + b' + (1-\delta)k$$

$$\mathcal{C}(k,k') = \frac{\psi(k'-k)^2}{2k} + \chi \mathbb{1}\{k' \neq (1-\delta)k\}$$

$$y = zk^{\alpha} n^{1-\alpha}$$

- Source of misallocation
  - Uncertainty of TFP process z: investment made this period but shock revealed next period
  - Convex cost ψ: smooth capital stock over time
  - Fixed cost χ: inaction of investment

Collateral Constraint (MX 2014 AER; GKKV 2017 QJE)

- Firm dynamic model

$$v(k,b,z) = \max_{p,n,k',c,b'} \left\{ u(c) + \beta \mathbb{E} v(k',b',z') \right\}$$

$$\text{s.t.} \quad c + k' + (1+r)b = py - wn + b' + (1-\delta)k$$

$$b' \leq \theta k'$$

- Source of misallocation
  - Collateral constraint θ: maximum borrowing proportional to firms' assets
  - Size-dependent collateral constraint

$$\theta(k) = \theta_0 + \theta_1 \frac{\exp(k) - 1}{k}$$

Growing Like China (SSZ 2011 AER)

- Two types of firms produce the same good:

$$Y_{jt} = A_j K_{jt}^{\alpha} L_{jt}^{1-\alpha}, \qquad j \in \{E, F\}, \qquad A_E > A_F.$$

  - Entrepreneurial firms (E): high productivity, but constrained.
  - Financially integrated firms (F): low productivity, but unconstrained
- Let R be the user cost of capital. Unconstrained F firms satisfy

$$\alpha A_F \left(\frac{K_{Ft}}{L_{Ft}}\right)^{\alpha-1} = R.$$

- Labor is mobile, so both types pay the same wage:

$$(1-\alpha) A_E \left(\frac{K_{Et}}{L_{Et}}\right)^{\alpha} = (1-\alpha) A_F \left(\frac{K_{Ft}}{L_{Ft}}\right)^{\alpha} = w_t.$$

- Consequently, while both types coexist,

$$\frac{\mathrm{MPK}_{Et}}{\mathrm{MPK}_{Ft}} = \left(\frac{A_E}{A_F}\right)^{1/(1-\alpha)} > 1.$$

Growing Like China (SSZ 2011 AER)

- Wealth accumulation relaxes financing constraints.

$$K_{Et} \leq \lambda a_{Et}, \qquad \lambda \geq 1$$

  E firms expand through wealth accumulation a_{Et}.

- Labor moves from F to E firms, raising aggregate productivity.
- Capital flow
  - Workers deposit savings with banks.
  - Shrinking F firms demand fewer bank loans.
  - Banks cannot channel these funds to constrained E firms.

$$\underbrace{B_t}_{\text{net foreign assets}} = \underbrace{D_t}_{\text{bank deposits}} - \underbrace{K_{Ft} + \left(1 - \frac{1}{\lambda}\right) K_{Et}}_{\text{domestic bank lending}}$$

- High domestic returns coexist with net capital outflows.

Entry Barriers and Firm Selection (BKS 2026 REStud)

- Each region j has M_j potential entrepreneurs and a fixed labor supply N_j. Capital is available at rental rate $R = r + \delta$.
- Firms produce a homogeneous good:

$$y = z^{1-\eta}\left(k^{1-\alpha}n^{\alpha}\right)^{\eta}, \qquad 0 < \eta < 1.$$

- **Operating profits before the fixed cost:**

$$\Pi_j(z) = \max_{k,n}\left\{(1-\tau_{yj})y - w_j n - (1+\tau_{kj})Rk\right\}.$$

A licensed firm operates if

$$\Pi_j(z) \geq \nu \quad \Longleftrightarrow \quad z \geq z_j^*.$$

- **Entry barrier:** only a fraction $1 - \psi_j$ receives a license
- Labor-market clearing determines the wage:

$$M_j(1-\psi_j)\int_{z_j^*}^{\infty} n_j(z) f(z)\, dz = N_j.$$

Entry Barriers and Firm Selection (BKS 2026 REStud)

- Lower entry barriers $\psi \downarrow$:
  - More entrepreneurs are allowed to operate.
  - Raises labor demand and equilibrium wage rate.
  - Higher wages make low-productivity firms unprofitable, increasing productivity.
- **Evidence from China:**
  - Entry barriers account for much of the regional dispersion and convergence in wages and TFP.
  - SOE downsizing reduces barriers to non-state firm entry.
- **Beyond HK:** distortions affect not only inputs across existing firms, but also which firms operate.

Misallocation in Agriculture (ABLR 2022 ECTA)

- **Institution: land allocation in rural China.**
  - Village authorities allocate land largely according to household size rather than farming ability.
  - Limited rental markets prevent productive farmers from expanding.

- **A simple farm problem:**

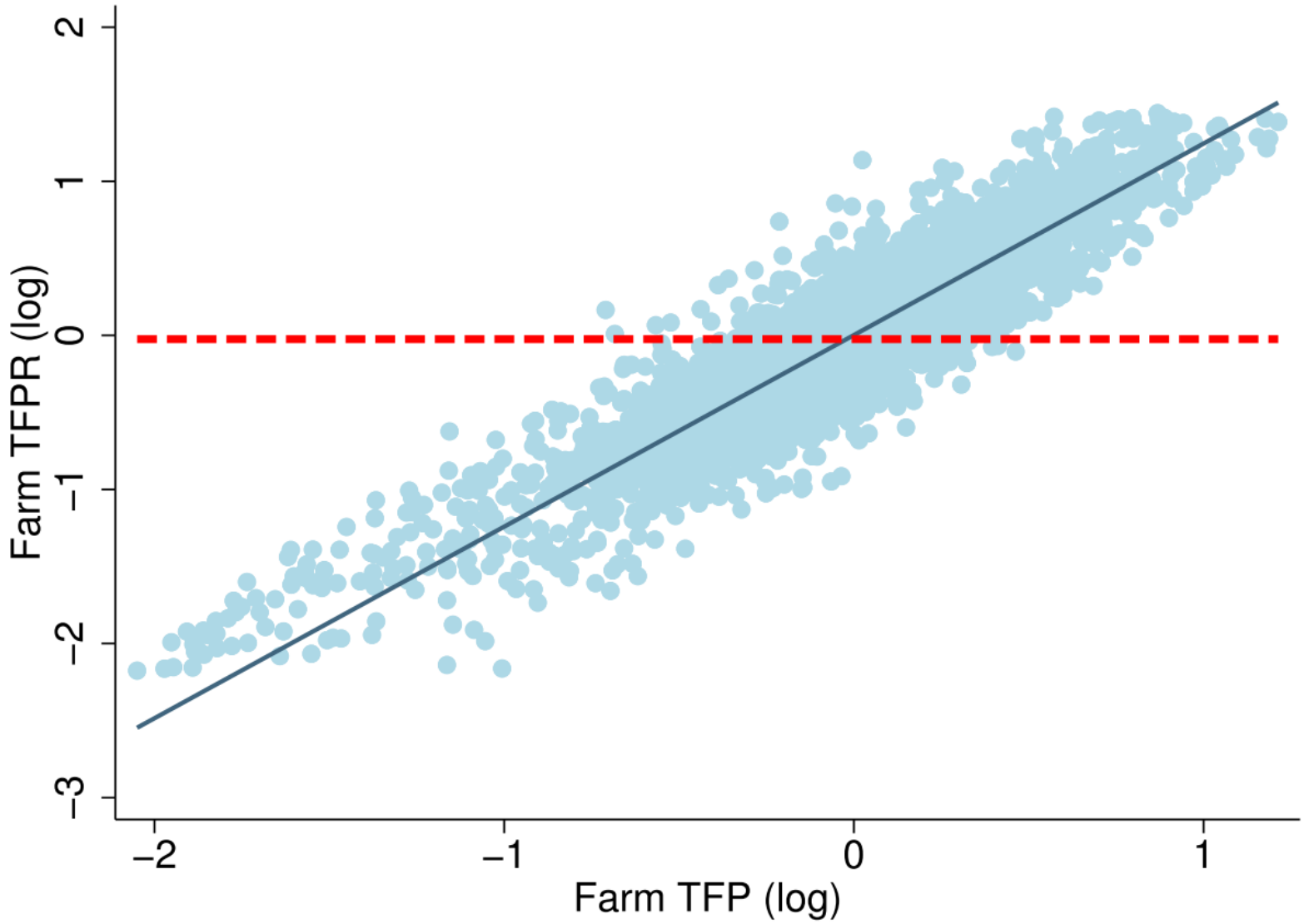
$$\pi_i^a = \max_{k_i, \ell_i} \left\{ (1-\tau_i) p_a A_i k_i^\alpha \ell_i^\beta - r_k k_i - r_\ell \ell_i \right\}, \qquad \alpha + \beta < 1.$$

  Here, ℓ_i denotes land and τ_i summarizes farm-specific distortions.

- The first-order conditions imply

$$p_a \mathrm{MPK}_i = \frac{r_k}{1-\tau_i}, \qquad p_a \mathrm{MP}_{\ell,i} = \frac{r_\ell}{1-\tau_i}.$$

Misallocation in Agriculture (ABLR 2022 ECTA)



Misallocation Also Distorts Occupational Selection

- Individuals differ in agricultural and nonagricultural ability, (A_i, z_i).
- **Occupational choice:**

$$\underbrace{\pi_i^a(A_i, \tau_i)}_{\text{farm income}} \geq \underbrace{w_n z_i}_{\text{nonfarm income}} \quad \Longrightarrow \quad \text{choose agriculture.}$$

- **Two effects of distortions:**
  1. **Misallocation:** productive farmers operate with too little land.
  2. **Selection:** distortions that disproportionately penalize productive farmers discourage them from choosing agriculture.
- Selection amplifies the productivity gains from better resource allocation.

Trade and Migration (TZ 2019 AER)

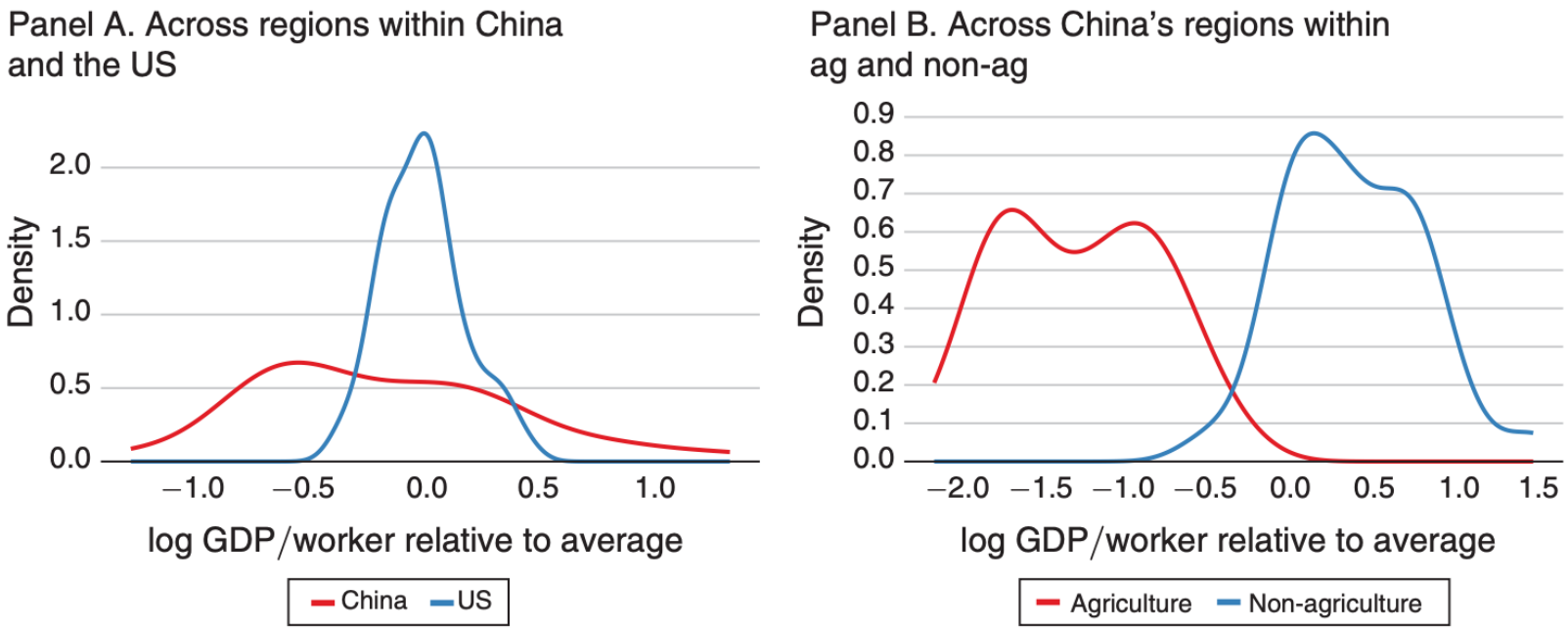


FIGURE 2. SPATIAL DISTRIBUTION OF GDP PER WORKER, 2000

Trade and Migration (TZ 2019 AER)

- Regions differ in productivity in agriculture and nonagriculture.
- **Goods face trade costs.** Delivering one unit from region i to region j requires shipping $\tau_{ij}^s \geq 1$ units:

$$p_{ij}^s(\omega) = \tau_{ij}^s c_i^s(\omega).$$

- **Workers face migration costs.** A worker h from origin o chooses a region-sector pair:

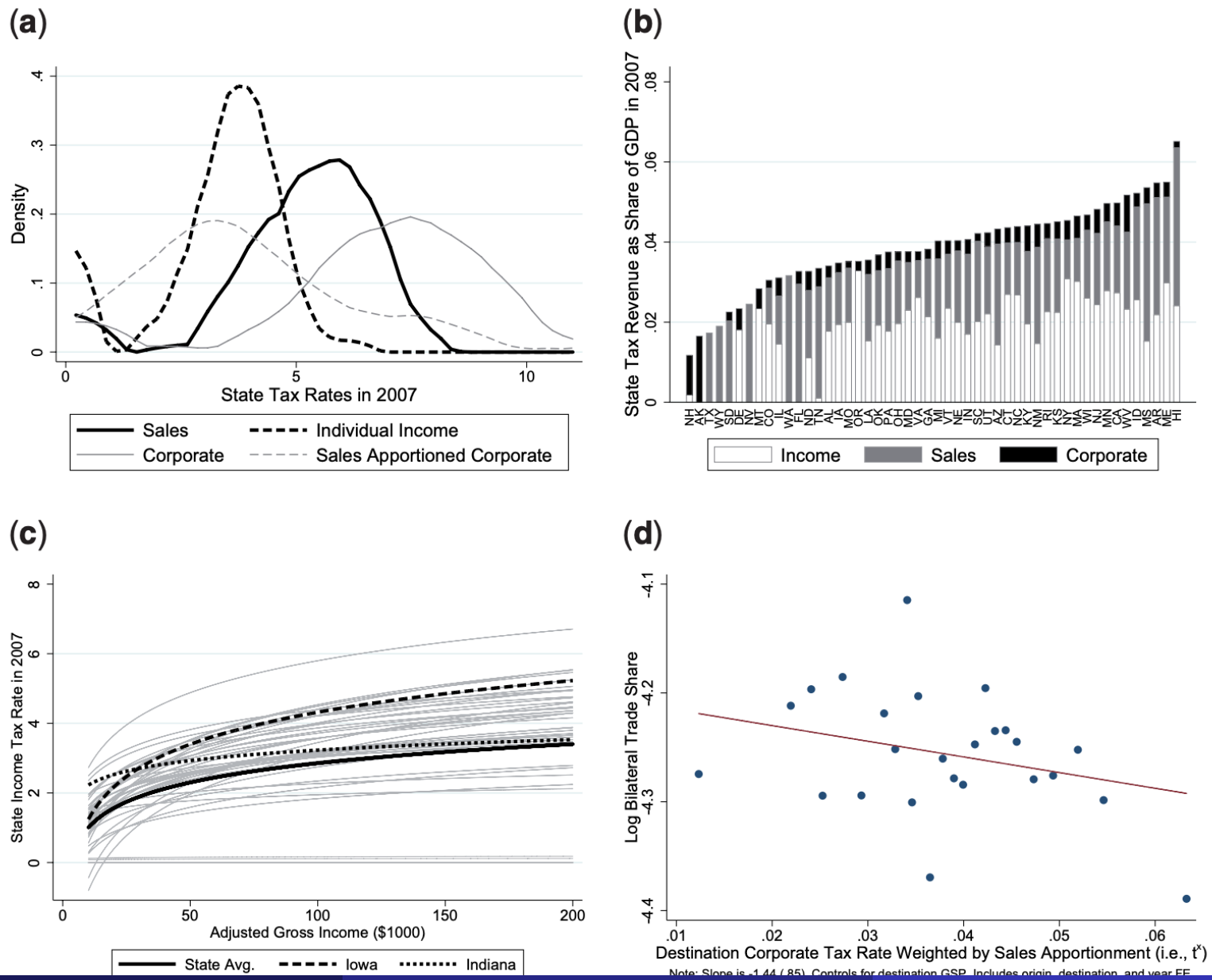
$$(j,s) \in \arg\max_{j,s} \left\{ \underbrace{\varepsilon_{hjs}}_{\text{preference shock}} \times \underbrace{\frac{w_{js}}{P_j}}_{\text{real wage}} \times \underbrace{\frac{1}{m_{o,js}}}_{\text{migration cost}} \right\}, \quad m_{o,js} \geq 1.$$

- Trade barriers limit specialization; migration barriers limit labor reallocation.

Trade and Migration (TZ 2019 AER)

- **Lower trade costs:**
  - Buyers obtain goods from lower-cost suppliers.
  - Production shifts toward regions with comparative advantage.
- **Lower migration costs:**
  - Workers move across regions and sectors.
  - Wages, prices, and employment adjust in general equilibrium.
- **Quantitative results for 2000–2005:**
  - Lower trade and migration costs account for 36% of aggregate labor productivity growth.
  - Internal trade and migration reforms account for 28%; lower international trade costs account for 8%.
- **Main message:** domestic market integration is an important source of productivity growth.

State Taxes


(a)
Density
0
.1
.2
.3
.4
0
5
10
State Tax Rates in 2007
Sales
Individual Income
Corporate
Sales Apportioned Corporate
(b)
State Tax Revenue as Share of GDP in 2007
0
.02
.04
.06
.08
NH AK TX WY SD DE NV MT CO IL WA FL ND TN AL IA MO OR OK PA OH MD VA GA MI VT NE NM SC UT AZ CT NC KY NM RI KS NY MA WI NJ MN CA WV ID MS AR ME HI
Income
Sales
Corporate
(c)
State Income Tax Rate in 2007
0
2
4
6
8
0
50
100
150
200
Adjusted Gross Income ($1000)
State Avg.
Iowa
Indiana
(d)
Log Bilateral Trade Share
-4.1
-4.2
-4.3
-4.4
.01
.02
.03
.04
.05
.06
Destination Corporate Tax Rate Weighted by Sales Apportionment (i.e., $\tilde{t}^x$)

State Taxes and Spatial Misallocation (FMSZ 2019 REStud)

- States differ in productivity, amenities, public services, and tax rates.
- Workers choose where to live and work:

$$j(h) \in \arg\max_j \left\{ \underbrace{\varepsilon_{hj}}_{\text{preference shock}} \quad \underbrace{A_j(G_j)}_{\text{amenities}} \quad \frac{\overbrace{(1-t_j^y)}^{\text{personal income tax}}}{\underbrace{(1+t_j^s)}_{\text{sales tax}}} \frac{w_j}{P_j} \right\}.$$

- Firms choose where to produce:

$$j(i) \in \arg\max_j \left\{ \underbrace{(1-t_j^c)}_{\text{corporate income tax}} \quad \pi_{ij}(G_j, w_j, \ldots) \right\}$$

State Taxes and Spatial Misallocation (FMSZ 2019 REStud)

- **Spatial allocation:** workers and firms respond to after-tax returns.
- **General equilibrium:** tax changes affect location choices, wages, prices, tax bases, and public spending.
- **Counterfactual: harmonize state tax rates.** Worker welfare increases by:

Government spending held fixed	0.6%
Government spending adjusts	1.2%

- **Important qualification:** uniform tax rates are not necessarily optimal. Welfare effects depend on amenities, productivity, and public spending.

Allocation of Talent (HHJK 2019 ECTA)

- Individuals differ in their talent across occupations:

$$\boldsymbol{\varepsilon}_h = (\varepsilon_{h1}, \ldots, \varepsilon_{hI}).$$

- **Human capital investment:** spending e produces human capital e^η, where $0 < \eta < 1$.
- A worker from group g chooses an occupation and investment:

$$\max_{i,e \geq 0} \left\{ \underbrace{(1 - \tau_{ig}^w) w_i \varepsilon_{hi} e^\eta}_{\text{net earnings}} - \underbrace{(1 + \tau_{ig}^h) e}_{\text{education cost}} \right\}.$$

  - τ_{ig}^w: labor-market barriers that reduce the return to work.
  - τ_{ig}^h: barriers that raise the cost of acquiring human capital.

- After optimizing investment, occupational choice satisfies

$$i^*(h, g) \in \arg\max_i \left\{ \frac{1 - \tau_{ig}^w}{(1 + \tau_{ig}^h)^\eta} w_i \varepsilon_{hi} \right\}.$$

Allocation of Talent (HHJK 2019 ECTA)

Hsieh, Hurst, Jones, and Klenow (2019, Econometrica)

- **Motivating fact:** white men accounted for 94% of doctors and lawyers in 1960, compared with 62% in 2010.
- **Key identifying assumption:** relative innate talent distributions across demographic groups remain stable over time.
- **Lower barriers raise output through:**
  1. Better matching of workers to occupations according to comparative advantage.
  2. Greater investment in human capital.
  3. Greater participation in market work.
- **Quantitative result:** improved talent allocation explains 20–40% of growth in aggregate market output per person, 1960–2010.
- **Main message:** barriers to opportunity can reduce aggregate output by preventing talent from reaching its best use.

Summary for Today

- An accounting framework for capital misallocation (Hsieh and Klenow 2009 QJE)
- Sources of misallocation
  - Mismeasurement or misspecification?
  - Dynamic inputs and adjustment costs
  - Collateral constraint
  - Entry barrier
  - Land policy
  - Trade cost
  - Migration cost
  - State taxes
  - Barrier to education and work
  - ...

Homework and Presentation

Choose one of the following options and present next class:

1. **Paper presentation.** Choose a paper that we introduced but did not discuss in detail. Read it carefully and explain its main question, framework, evidence, and contribution.
2. **An application.** Find data and use an accounting framework from today's lecture to study a specific form of misallocation. Explain your assumptions, findings, and limitations.

You are encouraged to combine both options: read a paper on a topic you find interesting and use it to motivate an empirical application or extension.

For example, read the paper on agricultural land misallocation in China and use data to explore a related question, such as the misallocation of urban land.